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Comparing the Effects of Algebra Tiles and Multiplication Frame on Basic Nine Pupils Performance in Binomial Expansion and Factorization of Trinomials

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Abstract

Aim: This study was conducted primarily to compare the effectiveness of two known teaching strategies used in teaching expansion of two linear binomials and factorization of quadratic trinomials. The study employed a quasi-experimental design.

Methods: The study was conducted in three junior high schools in the Upper East Region of Ghana, using 113 pupils as a sample size. A test was used to collect data for this study, and the data was analyzed using analysis of variance (ANOVA).

Results: It was found that the post-test mean score of pupils taught with the multiplication frame was significantly different from the post-test mean score of pupils taught with the algebra tiles and the conventional method, but there was however no significant difference in the mean scores of pupils taught with the algebra tiles and those taught with the conventional approach.

Conclusion: The study concluded that the multiplication frame/box method was more effective than the use of algebra tiles.

Recommendation: The study recommend multiplication frame/box method to be the choice in teaching expansion of two linear binomials and factorization of quadratic trinomials for maximum gains.

Keywords: *Algebra tiles, multiplication frame/box method, binomial expansion*.

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INTRODUCTION

Expansion of two linear binomials and factorization of quadratic trinomials are two sides of the same coin. These two concepts are central to the topic algebra. Algebra is defined as the "science of generalized computations" (Sfard, 1995 as cited in Vilakazi, 2021). Having a firm grasp of algebra is crucial for moving onto more advanced mathematics courses like geometry and calculus (Star et al., 2015). Since algebra is the bedrock of the mathematics studied in high schools, it is crucial to comprehending a wide variety of topics and concepts (Makonye, 2015). Vygotsky as cited in Vilakazi (2021) asserts that, "the student who had mastered algebra had attained 'a new higher plane of thought', a degree of abstraction and generalisation that changes the meaning of the previous arithmetic level".

Factorization of quadratic trinomial is the concept that some mathematics teachers and their students have been struggling with during the teaching and learning process. This statement is supported by De Lima and Tall (2006) when they asserted that the teaching and learning of algebra has long been seen as a source of difficulty and this have raised questions and concerns among educators in the field of mathematics. Also, Matzin and Shahrill (2015) study revealed that, grade seven mathematics students had poor grasp of basic algebraic knowledge and skills and also experienced difficulties in solving problems that involved manipulation of algebraic expressions and equations.

In Ghana, Chief Examiner report (2007) noted that problems involving expansion and factorization of expressions has become cyclical year after year. Majority of students exhibits lack of understanding of the concept of quadratic expressions and other algebra related concepts. Also, the Chief Examiner reports (2000, 2002 & 2004) indicated that students encountered challenges in handling and solving algebraic expressions related problems. Students committed mistakes in solving equations of the form; $ax^2 + bx + c = 0$. In addition, some Basic Education Certificate Examination candidates do not have strong grasp of basic factorization or simplification which is worrying (BECE Home Mock Chief Examiner, 2022). Again, Yahya and Shahrill (2015) identified four main mistakes that learners' make when learning factorisation of quadratic expressions. These include; mistakes in understanding factorising quadratic expression, mistakes in multiplying the factors, mistakes in adding integers and mistakes in the formation of solutions for factorisation. These challenges could be due to the fact that teaching methods used in teaching these concepts does not encourage or promote relational learning.

There exist several teaching strategies teachers can use to teach these concepts. For instance, the FOIL, multiplication-frame/box method or algorithm and the use of algebra tiles can be used to teach expansion of binomials. Also, the trial-and-error method, grouping method, cross multiplication, multiplication frame as well as the use of algebra tiles can be used to teach factorization of quadratic trinomials.

Though many methods are available for teaching expansion of binomials and factorization of quadratic trinomials, not all make sense to students hence mathematics teachers need to identify the strategy that makes more sense to students and the one that is considered most effective (Chua, 2017). As a result of this and coupled with the researchers quest to boost their students understanding and performances in expansion of two linear binomials and factorization of quadratic trinomials, the researchers conducted a simple survey on twelve (12) experienced mathematics teachers. The survey was to find out from the experience teachers the strategy they

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think and believe is the most effective for teaching expansion of two linear binomials and the factorization of quadratic trinomials. Seven out of the twelve responded that the use of concrete objects specifically algebra tiles is the most effective whilst the remaining five teachers said the use of multiplication frame. These two were probably the only approaches known or frequently used by these mathematics teachers.

The assertion of the seven teachers in favour of algebra tiles has the backing of Saraswati, Putri & Somakim (2016) study which revealed that algebra tiles could help students learn linear equations in one variable whilst that of the five teachers in favour of the multiplication frame is backed by Caylan (2018) whose study however discovered that, employing algebra tiles did not produce noticeably better results than the conventional method of instruction. Also, their views were in consonance with the Singaporean secondary school mathematics syllabus recommendation for the use of multiplication frame instead of algebra tiles in teaching these concepts (Ministry of Education, (Singapore), 2012).

When the responses of these 12 experienced mathematics teachers on the perceived most effective strategy for teaching expansion of two linear binomials and factorization of quadratic trinomials were shared with other mathematics teachers in the researchers' schools, it ignited a debate among them as to which of them is actually the most effective and that necessitated the conduct of this study. This study was therefore conducted to determine the most effective approach between the box/multiplication frame and the use of algebra tiles in teaching expansion of two linear binomials and factorization of quadratic trinomials. To successfully determine this, the null hypothesis was that there is no significant difference in the post-test mean scores of pupils taught with box/multiplication frame method, the conventional approach and the use of algebra tiles. This null hypothesis was tested at 95% confidence interval. using the analysis of variance test statistic.

METHODOLOGY

Research Design

This quantitative study employed a quasi-experimental design to help collect data from basic nine pupils in three intact classes. The data collected was then used to test the null hypothesis that there is no significant difference in the post-test mean scores of pupils taught with box/multiplication frame method, the conventional approach and the use of algebra tiles.

Sample and Sampling Techniques

Convenience sampling technique was used to select four schools that were eager to participate in the study. These four schools were given a pre-test to help decide on schools that should be included in the study. Three out of the four schools were found to have almost the same cognitive entry behaviour level hence were selected. The school that performed significantly different from the other three schools in the pretest did not take part in the study. The selected schools were then randomly assigned as school A, school B and school C. The study used third-year classes from the three selected Junior High Schools. Third-year classes were considered because the said content area is studied in year three. School A had a population of 33, school class B had population of 37 and school C had a population of 43 and that gave a total sample size of 113. School A was taught with algebra tiles, school B was taught with the conventional approach and school C was taught with the multiplication frame/ box method.

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Instrumentation

Tests (Pre-test and post-test) was used to collect data for the study. Each test consisted of six items and marked out of 30. Pupils were given 30 minutes for each test. Validity of the instruments were established through peer review while reliability was ensured using test-retest reliability method. The tests (both pre-test and post-test) were administered on two different occasions to the same participants during the pilot testing on an interval of two weeks each. The two scores from each test were then correlated to determine the reliabilities of the test. The Pearson product moment correlation coefficients of 0.73 and 0.78 were obtained for the pre-test and post-test respectively indicating that, the instruments were reliable (Bryman & Crammer, 2012).

Data Collection Procedure

Data was collected from the three schools in six weeks and analyzed using the Statistical Package for Social Sciences Version 20. The specific statistical tool used for the data analysis was the analysis of variance (ANOVA). This parametric statistical tool was used because the data satisfied the three basic assumptions of parametric test (normality, independence and homogeneity of variance) and was also intended to compare the means of three different groups of students.

Instructions on Algebra

The researcher used six weeks to teach expansion of two linear binomials and factorization of quadratic trinomials in all three schools. To ensure fairness in all schools, the same examples were used with the only difference being the approaches used. Both pre-test and post-test were administered within the six weeks period.

School A Lessons, Experimental Class 1 (Algebra Tiles)

Students were introduced to the various algebra tiles for them to familiarize themselves with it and how to use it in solving problems. Blue colour was used to represent positive tiles while red colour indicated negative tiles.

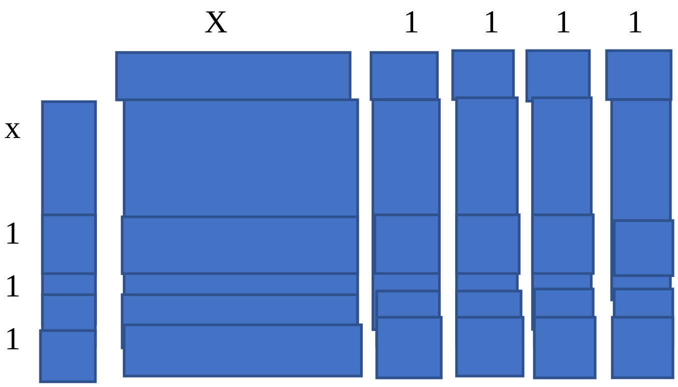

Figure 1. Expansion of two binomials using algebra tiles.

To expand the binomials (x+4) (x+3) as done in figure 1, students were guided to do the following;

a. Represent the factors (x+4) and (x+3) as the length and width of a rectangle.

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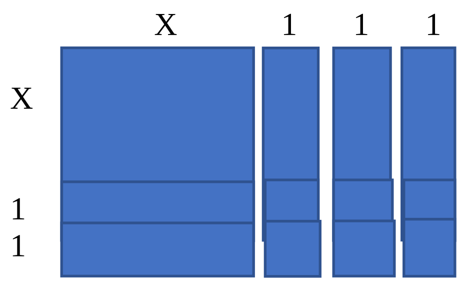
b. Lay out one X-tile and four unit-tiles to form (one side) the length of the rectangle, (x+4) as shown in figure 1.
c. Lay out one X-tile and three units-tiles to form (the other side) the width of the rectangle, (x+3) as shown in figure 1.
d. Lay out more materials (X-tiles and cubes) to complete a rectangle of sides (x+4) by (x+3) and exchange the required number of X-tiles for X by X-tiles as shown in figure 1.
e. Count the number of X by X-tiles, X-tiles and cubes or unit tiles that formed the rectangle.
f. Students notice that, there are 1(X by X-tile), 7 X-tiles and 12 units.
g. Students therefore conclude that, (x+4) (x+3) = X^2 +7X+12.

To factorize X^2+5X+6 using algebra tiles, students were taken through the following steps;

a. Identify the specified or given tiles in the question
b. Arrange the tiles to form a rectangle as shown in figure 2.
c. Students guided to note that, the length of the rectangle built is x+3 and the width is x+2
d. Students conclude that X^2+5X+6 = (x+3) (x+2)



X^2+5X+6 = (X+3) (X+2).

Figure 2. Factorization of quadratic trinomials using algebra tiles.

School B Lessons, Control Class (Conventional Approach)

In school B, students were taught using the conventional approach (FOIL and Grouping).

To expand (x+4) (x+3), the following activities were carried out;

a. Students identify the first terms and last terms of each binomial.
b. Students consider the two binomials as one entity and then identify the inside terms and outside terms.
c. Students find the product of the first terms as x^2
d. Students find the product of the outside terms as 3x
e. Students find the product of the inside terms as 4x
f. Students find the product of the last terms as 12
g. Add the resulting products as x^2+3x+4x+12 = x^2+7x+12.

To factorize X^2+5X+6 = ax^2+bx+c using the conventional approach, the following activities were carried out;

a. Students find the product ac as 1×6= 6
b. Express ac as multiple of two factors; 6 = 6×1, 6 = 3×2.
c. Sum each pair of the two factors; 6+1 = 7, 3+2 = 5.

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d. Identify the factors that gives a sum of b (5) and then replace b in the trinomial with those factors as; $X^2+2X+3X+6$
e. Group the new expression into two parts and find the common factor in each group. $(X^2+2X) + (3X+6) = X(X+2) +3(X+2)$
f. Group the outside terms together and pick one of the inside terms as the answer = $(X+3)(X+2)$.

School C Lessons, Experimental Class 2 (Box Method)

To expand (x+4) and (x+3) using the box method, the following steps were followed;

a. First draw a box and divide it into four equal parts.
b. Write the first binomial horizontally along the top of the box.
c. Write the second binomial vertically along the left side of the box.
d. Fill the top left box by multiplying the first terms of the binomials
e. Fill the top right box by multiplying the second term of the first binomial and the first term of the second binomial.
f. Fill the bottom left box by multiplying the first term of the first binomial and the second term of the second binomial.
g. Fill the bottom right box by multiplying the second terms of the two binomials.
h. Add the four terms together.

	X	+4
X	X^2	4X
+3	3X	12

$X^2+4x+3x+12 = X^2+7X+12$

Figure 3. Expansion of binomials using box method.

To factorize X^2+5X+6 using the box method, students were guided to do the following;

a. Draw a table and divide it into four equal parts.
b. Fill the top left box with the first term.
c. Fill the bottom right box with the last term (the constant).
d. Find the product of the coefficient of x^2 and the constant term (ac).
e. Find the factors of ac that add up to give the coefficient of X (b).
f. Fill the top right box and bottom left box with those factors.
g. Fill the top left box with the first term of the trinomial.
h. Fill the bottom right box with the constant term.
i. Find the factors of each of the four terms and write them as x +3 and x +2 as shown in figure 4.

	X	+3
X	X^2	3X
+2	2X	+6

Figure 4. Factorization of quadratic trinomial using box method.

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RESULTS AND DISCUSSIONS

The pretest results in table 1 was used to decide on the schools to be included in the study.

Table 1: Analysis of variance results for the pretest

School	M	SD	F	DF1	DF2	Sig.
First School	6.60	1.90	30.70	3	136	.000
Second School	11.07	2.30				
Third School	6.76	1.92				
Fourth School	6.54	2.77				

Source: Field data (2023).

The mean score and standard deviation for pupils in the first school is (M = 6.60, SD = 1.90), the second school is (M = 11.07, SD = 2.30), the third school is (M = 6.76, SD = 1.92) and that of the fourth school is (M = 7.49, SD = 2.77). A statistically significant difference was found among the four Junior High School pupils' performances; $F(3, 236) = 30.70$, $P < .001$. As a result, a post hoc test was conducted to determine where the difference lies and the post hoc test revealed that the difference was in favour of the second school. However, there was no statistically significant difference in the mean scores among pupils in the first, third and fourth schools. This provided the basis for including these schools in the study since the researchers wanted to have only pupils of the same cognitive entry behaviour in the study. Table 2 provides the post hoc Turkey HSD results for the pretest.

Table 2: Post -hoc test for pretest

(I) Schools that took part in the pretest	(J) Schools that took part in the pretest	Mean difference (I-J)	Std. Error	Sig.
First School	Second School	-4.46942*	.53157	.000
	Third School	-.15292	.50101	.990
	Fourth School	.06411	.48544	.999
Second School	First School	4.46942*	.53157	.000
	Third School	4.31650*	.56178	.000
	Fourth School	4.53353*	.54795	.000
Third School	First School	.15292	.50101	.990
	Second School	-4.31650*	.56178	.000
	Fourth School	.21704	.51835	.975
Fourth School	First School	-.06411	.48544	.999
	Second School	-4.53353*	.54795	.000
	Third School	-.21704	.51835	.975

Source: Field data (2023).

After pupils in the three schools had gone through algebra instructions using the specified approaches, they were examined again and their scores analyzed using ANOVA. Table 3 presents the ANOVA results for the posttest.

Table 3: Analysis of variance results (post-test)

School	M	SD	F	DF1	DF2	Sig
A	13.36	3.22	15.88	2	110	.000
B	13.47	2.60				
C	17.03	3.75				

Source: Field data (2023).

A one-way analysis of variance (ANOVA) was conducted to compare the effect of two teaching approaches (the use of algebra tiles and the use of multiplication frame also known as box method) against the conventional teaching approach on Junior High School pupils' performances in the expansion of two linear binomial expressions and factorization of quadratic trinomials. The mean score and standard deviation for students who were taught using algebra tiles is (M = 13.36, SD = 3.22), those who were taught using the conventional approach is (M = 13.47, SD = 2.60) and their counterparts taught using the box method is (M = 17.03, SD = 3.75). A statistically significant difference was found among the teaching approaches on Junior High School students' performances, $F(2, 110) = 15.88$, $P < .001$. This provided enough evidence to reject the null hypothesis which stated that, there was no significant difference in the posttest mean scores among the three groups of pupils taught using different approaches.

Table 4: Post Hoc test results for the Post-test

	(I) Method of teaching	**(J) Method of teaching**	**Mean difference (I-J)**	**Std. error**	**Sig.**	**95% Confidence interval**	
						Lower bound	**Upper bound**
Tukey HSD	Algebra	Box	-3.663^{*}	.764	.000	-5.48	-1.85
		Conventional	-.101	.739	.990	-1.86	1.65
	Box	Algebra	3.663^{*}	.764	.000	1.85	5.48
		Conventional	3.562^{*}	.716	.000	1.86	5.26
	Conventional	Algebra	.101	.739	.990	-1.65	1.86
		Box	-3.562^{*}	.716	.000	-5.26	-1.86

**The mean difference is significant at the 0.05 level.*

Source: Field data (2023)

The ANOVA results only showed a significant difference among the teaching approaches on Junior High School pupils' performances in the expansion of two linear binomial expressions and factorization of quadratic trinomials but could not show where the difference(s) lies. As a result, a post hoc Tukey HSD was conducted to evaluate the pairwise differences among the means of the three groups of students. The result revealed that, the box method (n = 37, M = 17.03, SD = 3.75) was significantly different from the conventional method (n = 43, M = 13.47, SD = 2.60) and the use of algebra tiles (n = 33, M = 13.36, SD = 3.22).

However, there was no significant difference in the mean scores between students taught using the conventional teaching approach and those taught using the algebra tiles. It therefore means that, the box method has a positive impact on Junior High School pupils' academic attainment than the conventional teaching approach and the use of algebra tiles. This suggests that pupils whose teachers employ the box method as a teaching strategy in teaching expansion of two linear binomial expressions and factorization of quadratic trinomials are more likely to perform better than those whose teachers employs either the conventional approach or the use of algebra tiles.

The finding of this study is backed by Caylan (2018) who discovered that, employing algebra tiles did not produce noticeably better results than the conventional method of instructions but however contradicts that of Saraswati, Putri & Somakim (2016) whose study found the use of algebra tiles to be effective in teaching linear equations in one variable. It also supports the recommendation made by the Singapore secondary school syllabus for the Singapore mathematics teachers to be using the box method/multiplication frame in teaching expansion and factorization (Ministry of Education, [MOE], 2012).

CONCLUSIONS AND RECOMMENDATIONS

From the testing of the hypothesis, it was found that, there was a significant difference in the post-test mean score of pupils who were taught expansion of two linear binomials and factorization of quadratic trinomials using the teaching approaches. Pupils who were taught with the box method performed significantly better than their counterparts who were taught with the algebra tiles and the conventional teaching approach.

Also, there was difference in the mean scores between students who were taught using the algebra tiles and those who were taught using the conventional teaching approach but the difference was not statistically significant. For these reasons, the researcher concluded that, the most effective approach between the two approaches for teaching expansion of two linear binomials and factorization of quadratic trinomials is the Box method/multiplication frame. This study recommends the use of Box method/multiplication frame in the Ghanaian schools especially in the Upper East Region where mathematics teachers were contemplating on which approach is considered the most effective. Also, curriculum developers should ensure that, the multiplication frame otherwise known as the box method forms an integral part of the Basic and Senior High Schools mathematics curriculum.

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Conflict of interest

The authors declare that there is no conflict of interest regarding its publication.

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