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Pre-service Mathematics Teachers' Errors and Misconceptions of Integer Operations

Alexander Owusu[1], Christopher Saaha Bornaa[2], Dennis Offei Kwakye[2*], Abdulai Boare Iddrisu[3]

[1]Department of Mathematics/ICT, St. Louis College of Education, Kumasi, Ghana

[2]Department of Mathematics and ICT Education, C. K. Tedam University of Technology and Applied Sciences, Navrongo, Ghana

[3]Department of Mathematics, Kalpohin Senior High School, Tamale, Ghana

[*]Corresponding Author's Email: dokwakye@cktutas.edu.gh

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Abstract

Purpose: This study investigated common errors and misconceptions made by some pre-service mathematics teachers of the St. Louis College of Education in Ghana about operations on integers. Errors associated with the misconceptions were also explored.

Methodology: The study employed mixed-method and sequential explanatory design, Diagnostic test instruments were designed and administered to sixty (60) pre-service mathematics teachers who offer specialized programmes in early grade (EG) and upper primary (UP).

Findings: The study identified common errors made by pre-service mathematics teachers as interpretation of the inequality symbols $'<'$ and $'>'$, addition and subtraction of integers, multiplication and division of integers, problem-solving (word problems). The study also found introduction concept of operations on integers, interpretation of absolute value of integers, problems involving change in the order of operation of integers as common errors made by pre-service mathematics teachers. Furthermore, the study found problems involving more than one operation, over-reliance on calculator usage and rote-learning, and inability to formulate word problems as mathematics problem.

Conclusion: The study conclude that deficiency in the language was the course of their inability to solve word problems.

Recommendations: The study recommends that when solving word problems involving integers, pre-service mathematics teachers should take read over the problem thoroughly to understand the problem properly. Also, during teaching and learning of mathematics, particularly operations on integers, much attention must be given to helping learners to translate normal language into mathematics language to overcome language deficiency. The study suggest that similar study could be done in other Colleges of Education and institutions to expound the findings.

Keywords: *Errors, negative numbers, operations on integers, positive numbers, ordering of integers, pre-service mathematics teachers' errors, pre-service mathematics teachers' misconceptions*

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INTRODUCTION

Mathematics is an essential part of people's daily lives and it is a universal that it is necessary for development. Also, mathematics is a body of information that attempts to explain and interpret phenomena and experiences. Mathematics has changed human lives and is critical to Ghana's future growth (NaCCA, 2019). The importance of mathematics education is critical to the development of any society. However, many children develop a negative attitude toward mathematics because they have a misunderstanding of what mathematics is and how important the subject is in their daily lives (Gyampoh, Nyarko & Agyeman, 2020).

The term integer is derived from the Latin word "integer," which means "whole" or "complete." Integers are a type of number that consists of zero, positive numbers, and negative numbers (Kwakye et al., 2022). An integer is a set of numbers which consists of a set of positive whole numbers, zero and the set of negative whole numbers (Bishop, Lamb, Philipp, Whitacre, & Schappelle, 2013; Kwakye et al., 2022; Ramani & Siegler, 2008). The integers can be shown on a number line that extends endlessly in both directions from zero. Every integer represents a magnitude and direction. Magnitude is the number of units the integer is away from zero on the number line. Direction can be positive (usually to the right on a number line) or negative (usually to the left on a number line). Positive can be any direction as long as negative is the opposite direction. Here are some examples of magnitude and direction; "3" has magnitude "3" and positive direction, and "-5" has magnitude "5" and negative direction (Gyampoh et al., 2020, Kwakye, et al., 2022).

Before they are formally introduced in school, young children are frequently exposed to negative numbers in contexts such as negative temperatures, negative points in golf or video games, or when selecting floors below zero in elevators in some countries (Bofferding & Hoffman, 2014). Understanding negative numbers can help students understand that zero is not the smallest number, which is a misconception they struggle to overcome. Furthermore, having experience counting backward through zero can help them understand that expressions like $3-5$ are meaningful, making the misleading rule that you cannot subtract a larger number from a smaller one unnecessary. There is, however, little research on how children's informal experiences with negative numbers may influence how they interpret these numbers. To include negative integers in children's mental number lines, they must extend the number sequence to the left of zero (or less than zero), with numerals symmetric to positive ones but marked with negative signs. They must also extend the concept that numbers further to the left on their mental number lines correspond to smaller values, such as "-5" being less than "-3" (Bofferdsing & Hoffman, 2014). Studies show that in recent times a lot of models have been developed for the teaching and learning of operations on integers (Cetin, 2019; Kwakye, et al., 2022; Stephan & Akyuz, 2012).

Integers form an integral part of the mathematics courses for the pre-service teachers of the Colleges of Education in Ghana. The mathematics courses for the pre-service teachers of the upper primary and the early grade specialisms are fashioned in line with the content of the new mathematics curriculum for the primary school. There are courses in both content and methodology in the mathematics for the pre-service teachers of the Colleges of Education which require the knowledge of integers to understand. However, previous studies have shown that mastering integers is incredibly challenging, presenting students with a range of misunderstandings that can last well into their adult ages (E. g. Bofferdsing & Hoffman, 2014).

Students' misconceptions were neither inborn nor instantaneous instead, students picked up certain views in the course of their process of learning for unknown reasons. Whatever the source, a way to recognize and resolve the problem should be available (Tiwari & Fatima, 2019).

Although operations on integers are introduced at the Lower Primary School and Junior high school levels, some students at Even College of Education level have difficulties in understanding integers because they find it more abstract than any other field of mathematics (Fuadiah, Suryadi, & Turmudi, 2016). Fadillah and Susiaty (2019) asserts that integer operation is an important concept in mathematics learning in lower secondary school yet, many students still have misconceptions in doing integer operations. According to Tiwari and Fatima (2019), some errors are persistent, so that they will occur due to misconceptions which are open to analysis, rather than the random errors that merely occur due to human fallibility. Therefore, examining deep into student thinking and their beliefs is necessary to find reasons for them to make these misconceptions. Since student reasoning is based on student beliefs for their errors, one can argue that these beliefs are wrong because they led to incorrect mathematical results.

Misconceptions, according to Bursal (2012) are caused by poor reasoning, thus, as people try to interpret the world around them, they form misconceptions. Bursal identified the following as some of the sources of misconceptions: Nonscientific beliefs, such as legendary teachings, entail attachments to scholarly articles other than scientific training; informal concepts produced in ordinary life, particularly via the abuse of language (e.g., the utilization of terminologies sun rise, sun set) and Incomplete or incorrect viewpoints developed by students during classroom instruction.

Misconception, in a broad sense, is the misinterpretation of knowledge, either directly or indirectly. Misconceptions are described as knowledge that prevents a person from learning scientific truths and is acquired through personal experiences. Misconceptions are false thoughts or beliefs that a person accepts as true and practices as a habit. Whenever learners learn a different topic, it's vital that they clear up any misconceptions they have about the material (Ozkan & Ozkan, 2012). A recent study on elementary school teachers indicated that the students had a weak foundation because the previous teachers did teach inappropriate strategies (Seifi, Haghverdi & Azizmohamadi, 2012). Before introducing a new concept to students, it is important for students to correct their misconceptions on the concepts underlying the new concepts that will be learned. It is also essential for the teacher to distinguish whether students have misconceptions or not understanding the concept. Students with misconceptions offer a teacher some benefits to decide whether they are correct or incorrect by pointing out their misconception from one concept to another. Students' awareness of their misunderstanding is the foundation to check their misconceptions (Fadillah & Susiaty, 2019).

Onder, Senyigit and Silay (2017) found exaggerated generalizations, incorrect terminology used in ordinary speech, and misconceptions stemming from prior learning experience as sources of pre-service mathematics teachers' misconceptions. They also observed that pupils who held these misunderstandings had difficulty completing a task. When teachers deliver instruction on topics in various subjects, as per Lucariello (2014), they are instructing pupils who also have some pre-instructional information about the topic. However, student knowledge can be incorrect, irrational, or misleading. Misconceptions are seen as steps that school children must take, steps that must be managed from an instructional point of view, and steps that are not an obstacle to participants' future education if they are based on weak and fragile pictures of a notion; they are an obstacle to gaining knowledge if they are based on stable and strong frameworks of a notion. Misconceptions are pictures which formerly functioned well but are now incorrect in a new setting, necessitating a broadening of the concept's application (Jabal & Rosjanuardi, 2019).

Many studies have revealed the most of the difficulties student's experience with operations on integers are as a result of the misconceptions they have from their previous learning experiences both at home and from their previous classes or school levels (e. g. Rosyidah, Maulyda, Jiwandono, Oktaviyanti, & Gunawan, 2021). As a result, the focus of this research is on the errors and misconceptions of pre-service mathematics teachers on operations on integers.

Problem Statement

Even though there are a lot of research on misconception of some basic concepts in mathematics, there is no enough studies that presents misconceptions of integers in the Colleges of Education in Ghana. Consequently, this study is set to investigate the common errors made by the pre-service teachers of the Colleges of Education in the Kumasi metropolis about operations on integers, find out the misconceptions associated with the errors and offer suggestions that could be used to help pre-service teachers to overcome their difficulties.

Research Questions

The following research questions guided the conduct of this study:

1. What are the common errors made by the pre-service mathematics teachers on operations on integers?
2. What misconceptions may account for the erroneous answers of pre-service mathematics teachers on operations on integers?
3. Which of the specialized pre-service mathematics teachers are more likely to commit errors on operations on integers?

LITERATURE REVIEW

The Concept of Integers

Integers are taught and learned primarily in Grades four, five, and six (Stephan & Akyuz, 2012; NaCCA, 2019). Poor arithmetic computation (without use of calculators) which includes operations on integers is one of the weaknesses identified in candidates' work by the chief examiner's report (West African Examination Council (WAEC), 2013). The report further indicated that very good attempt was made by candidates at solving problems on inequalities though few ignored the reversal of the inequality signs (WAEC 2013, BECE, 2013). According to reports by chief examiners reports (WAEC 2013, BECE, 2013), students are particularly devoted to their misconceptions because they actively perceived them during their experiences. Students' misconceptions regarding integer addition and subtraction must be rectified since the topic is a prerequisite for mastering advanced mathematics topics such as algebra. When misunderstandings exist regarding the prerequisite concepts, the absorption and incorporation of new information would be impeded (Fadillah & Susiaty, 2019). A lot of studies about misconceptions of mathematics concepts and their implications toward the learning of new concept appears to suggest that misconceptions serve as barriers to the formation of new concepts by learner (Bursal,

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2012; Jabal & Rosjanuardi, 2019; Jankvist & Niss, 2018; Lucariello, 2014; Permata & Wijayanti, 2019).

Observations at the College of Education under various programmes (including Teachers Certificate 'A', Diploma in Basic Education, and Bachelor's degree in Basic Education) has noted that a majority of pre-service mathematics teachers finds it difficult to solve problems involving integer operations and, as a result, rely on calculators to solve them. However, methodology problems of operations on integers demand a step-by-step full description of how the activities are carried out, and as a result cannot be solved by the use of calculators. Many pupils give incorrect answers to problems involving integers, indicating that they have misunderstandings about integers. Students are frequently perplexed by the positive and negative sounds that are inherent in integer operations. According to Embong (2020), students' lack of understanding of integers leads them to struggle while tackling mathematics problems, especially those that involve the four basic operations.

RESEARCH METHODS

The study employed a mixed-method and sequential explanatory design that distinguishes itself by collecting and analyzing quantitative data before collecting and analyzing qualitative data. The data was analyzed using percentage errors and mean percentage errors. The mean percentage error was interpreted as follows: 0.00 – 20.00 (Very Low); 20.01 – 40.0 (Low); 40.01 – 60.00 (Average); 60.01 – 80.00 (High); 80.01 – 100.00 (Extremely High). The population of the study comprised the pre-service teachers of St. Louis College of Education in Kumasi. A total of 60 Trainees selected from the early grade specialism and the upper primary specialism formed the study sample at this stage. This method was considered most appropriate at this stage since it allowed for the selection of trainees from each of the programme specialisms which is in line with the intention of studying the misconceptions of the trainees across each of the programme specialisms.

RESULTS AND DISCUSSION

General Performance of Pre-Service Teachers on Operations on Integers

Table 1 shows that majority of the pre-service teachers scored very low marks on problem solving involving more than one operational sign and ordering of integers in order of magnitude, 3.33% and 16.67% respectively. Again, pre-service teachers' performance on interpretation of absolute value of integers and problem solving (word problems) representing 10.00 % and 33.33 % respectively is not encouraging. However, the pre-service teachers performed relatively better in addition, subtraction, multiplication and division (themes two and three) with mean percentages of 72.67% and 66.25% respectively. Overall mean percentage score of 36.18% is an indication that the Pre-service teachers' performance on operations on integers was not encouraging. This weak percentage score in performance suggests that the pre-service teachers have a lot of misconceptions with operations on integers.

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Table 1: General performance in operations on integers

S.No	Themes	Mean percentage
A	Interpretation of the inequality symbols $'<'$ and $'>'$	37.00%
B	Addition and subtraction of integers	72.67%
C	Multiplication and division of integers	66.25%
D	Problem-Solving (word problems)	33.33%
E	Interpretation of absolute value of integers	10.00%
F	Problems involving change in the order of operations of integers	54.17%
G	Solving problems involving more than one operation	3.33%
H	Ordering a set of integers	16.67%
	Overall performance	**36.18**%

Errors Based on Themes

Table 1 shows the errors of pre-service teachers based on themes. Pre-service teachers' errors were classified into the following themes to ensure better analysis: (A) errors in interpreting the inequality symbols $'<'$ and $'>'$, (B) errors in addition and subtraction of integers, (C) errors in multiplication and division of integers, (D) errors in problem solving (word problems), (E) errors in interpretation of absolute value of integers and (F) errors in problems involving change in the order of operation, (G) errors of solving problems involving more than one operation, and (H) errors of ordering a set of integers. The mean percentage error – 80.00 (High); 80.01 – 100.00 (extremely high).

A. Errors of interpreting the inequality symbols '< and >'

Table 2 shows the mean percentage errors in the theme, "interpretation of the inequality Symbols < and >.

Table 2: Mean percentage errors in the interpretation of the inequality symbols '< and >'

Question number	Test item	Number of incorrect responses	Percentage error	Means percentage error
Question 1	Fill in the box with any of the inequalities $(>)$ or $(<)$ to make this expression true: $-12 \square 8$	28	46.6%	
Question 2	List three integers that can appropriately take the place of y, if $y < -5$	47	78.33%	
Question 3	List three integers thar can appropriately take the place of k, if $-10 < k$	38	63.33%	62.77%

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In Table 2, test items 1, 12, and 15 were intended to find out how the pre-service teachers could interpret the inequality symbols '< and >'. Question 1 was intended to find out how pre-service teachers could place the correct inequality symbol between the two numbers -12 and 8 to make a true statement. It could be seen from Table 2 that, twenty-eight (28) out of the sixty (60) pre-service teachers representing 46.67% could not complete the integer sentence with the use of an appropriate inequality symbol. For questions 12 and 15, pre-service teachers were made to select 3 sets of numbers (integers) each that could appropriately be used to replace the variables 'y' and 'k' in the algebraic expressions $y < -5$ and $-10 < k$ respectively. About 47 out of 60 and 38 out of 60 of the pre-service teachers respectively failed to solved $y < -5$ (Question 12) and $-10 < k$ (Questions 15). The percentage errors of 78.33% and 63.33% as indicated in Table 2 shows that most of the pre-service teachers have difficulties in interpreting the inequality symbols.

A critical look at the solutions of some of the pre-service teachers' scripts for question 12 shows that instead of listing a set of integers less than -5 (for the case of $y < -5$), $\{5, 6, 7, \ldots.\}$ were rather listed by the pre-service teachers. Also, for the expression '$-10 < k$', some pre-service teachers listed down integers such as {-11, -12, -13….} which are less than -10 as an answer instead of listing down integers which are rather greater than -10. The interview with some of the pre-service teachers further revealed that some of the basic school teachers use their left upper and the lower arms to introduce the concept of less than, and right upper and lower arms to introduce greater than thus making students at that level to belief that '$<$' for less than and '$>$' for greater than. It is however interesting to note that these inequality symbols '$<$ and $>$' do not necessarily mean "less than" and "greater than" as pre-service teachers were made to believe. For example, $-12 < -8$ may either be interpreted as "-12 is less than -8" or "-8 is greater than -12" which indicates that the same symbol may mean either "greater than" or "less than". The mean percentage error of 62.77% for the three test items on trainee's interpretation of the inequality symbols couple with the explanations from the interview as explained above affirms the fact that pre-service teachers carry along with them a lot of misconceptions on operations on integers from the basic schools they attended.

Hattikudur and Alibali (2010) pointed out that students might have grossed over the crucial feature of the inequality sign in expressing the relations between quantities at the elementary school when the concept was introduced to them. This assertion is confirmed by the pre-service mathematics teachers' difficulties in interpreting the inequality symbols as shown in the abysmal performance of pre-service mathematics teachers under theme one. Pre-service mathematics teachers' inability to interpret the inequality symbols is attributed to misconceptions they carry from the basic school as a result of how the inequality symbols were introduced to them. Again, the misconception of pre-service mathematics teachers with inequality symbols as a result of two different interpretations from the same inequality sign (for either greater than or less than) is in accordance with the findings Rosyidah et al., (2021). Of different settings, the symbols in algebra have varied meanings and interpretations. Students' impressions of letters, numerals, and symbols are erroneous and incomplete. They came to the conclusion that children struggle to understand how letters and signs are used in varied scenarios

B. Errors in the addition and subtraction of integers

Table 3 shows the mean percentage on errors of pre-service mathematics teachers on addition and subtraction of integers.

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Table 3: Mean percentage of pre-service mathematics teachers' errors in addition and subtraction of integers

Question number	Test item	Number of incorrect responses	Percentage of error	Means percentage error
Question 2	Evaluate $1-(-1)$	12	20.00%	
Question 3	Evaluate $-3+5$	19	31.67%	
Question 5	Evaluate $+3-(-4)$	18	30.00%	27.22%

In Table 3, questions, 2, 3, and 5 were formulated to find out how pre-service teachers could solve problems involving operations on addition and subtraction of integers. Table 3 shows twelve (12) p re-service teachers representing 20.00% of the total pre-service teachers sampled failed to answer q uestion 2. The question 2 had the question "Evaluate $1-(-1)$. In question 3, nineteen (19) pre-service teachers representing 31.67% could not provide correct answer to the question, "Evaluate $-3+5$" and eighteen (18) of the Pre-service teachers (30.00%) wrongly answered question 5, "Evaluate $+3-(-4)$". Again, from Table 3, it appears $20.00\%, 31.67\%$ and 30.00% errors recorded by the pre-service teachers with regards to addition and subtraction of integers were on the low side. However, a closer and critical look at the mean percentage error of 27.22% sends a worrying signal that needs critical attention. The respondents are pre-service teachers who are supposedly undergoing training to be equipped with the knowledge and skills needed to teach the young ones the operations on integers. For such pre-service teachers who are supposed to be master to harbor misconception about the subject matter to the level as much as 27.22% implies there is a course for concern. The obvious question that needs an immediate answer is "*How effectively can a teacher with such misconceptions about operations on integers help learners to perform operations on integers*"?

Wrong answers provided by some pre-service teachers to questions 2, 3, and 5 are summarized in Table 4. For example, some pre-service teachers obtained 0 and -2 for "$Evaluate\ 1-(-1)$", others too obtained $-8, 8, -2,$ and 2 for "$Evaluate\ -3+5$" whereas some had 1, -1, and -7 as answers to "*Evaluate* $+3-(-4)$". The assertion by Fuadiah, Suryadi and Turmudi (2019) that classroom learning analysis of the concept taught to students does not provide a clear explanation of minus in subtraction operations and as a negative number marker and that teaching also relies on the use of practical rules or formulas, which were intended to help students but ultimately led to their confusion is confirmed the answers provided some pre-service mathematics teachers. For example, some pre-service mathematics teachers obtained 0 and -2 for "$Evaluate\ 1-(-1)$", others too obtained $-8, 8, -2,$ and 2 for "$Evaluate\ -3+5$" whereas some had $1, -1$ and -7 as answers to "$Evaluate\ +3-(-4)$".

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Table 4: Wrong answers to the addition and subtraction of integers

Question number	Statement	Wrong answer provided
Question 2	Evaluate $1-(-1)$	0,
		-2
Question 3	Evaluate $-3+5$	-8,
		-2,
		2
		8
Question 5	Evaluate $+3-(-4)$	-1,
		1,
		7

C. Errors in the multiplication and division of integers

Table 5 show errors in the multiplication and division of integers by pre-service mathematics teachers.

Table 5: Errors in the multiplication and division of integers by pre-service mathematics teachers

Question number	Test Item	Number of incorrect responses	Percentage error	Means percentage error
Question 4	Evaluate $(-8)\times$	10	16.67%	
Question 6	Evaluate $-3\ \times$	32	53.33%	
Question 13	Evaluate $12\ \div\ (-2)$	26	43.33%	
Question 14	Evaluate $-20\ \div\ 2$	18	30.00%	35.83%

In Table 5, questions 4, 6, 13 and 14 were intended to explore whether or not the pre-service teachers have misconceptions regarding multiplication and division of integers. The results as indicated in Table 5 shows that ten (10) pre-service mathematics teachers with percentage error of 16.67%. Thirty-two (32) pre-service mathematics teachers with percentage error of 53.33%, twenty-six (26) pre-service mathematics teachers with percentage error of 43.33% and eighteen (18) pre-service mathematics teachers with percentage error of 30.00% respectively failed to answer the following questions: "$Evaluate\ (-8)\times(+2)$"; "$Evaluate\ -3\times(-5)$"; "$Evaluate\ 12\div(-2)$" and "$Evaluate\ -20\div 2$". By comparing the percentage errors of the questions of this category, it is clearly observed that pre-service mathematics teachers have misconceptions with multiplication and division of integers. The mean percentage error of 53.33% which is the highest in this category shows that the misconception of the pre-service mathematics teachers is much higher when dealing with multiplication involving two negative integers than when a positive and a negative integer are involved. Again, a low percentage error of 16.67% indicates that the pre-service mathematics teachers perform better when multiplying a positive integer by a negative integer. The percentage error of 43.33% and 30.00% for questions 13, "*Evaluate* $12 \div (-2)$" and question 14, "*Evaluate* $-20 \div 2$" make it clear that pre-service mathematics teachers experience much difficulties in dividing a positive integer by a negative as compared to dividing a negative integer by a positive integer. Both questions 4 and 14 were intended to find out how pre-service mathematics teachers perform operations of multiplication and division involving a negative number and a positive number. Ten (10) pre-service mathematics teachers representing 16.67% answered question 4 wrongly whereas eighteen (18) of them, 30.00%, answered question 14 wrongly. Table 5 shows that though qre-service teachers have difficulties with both situations (multiplication of a negative number by a positive number and division of a negative number by a positive number), their difficulty appears to be much greater with division of a negative number by a positive number.

D. Errors on problem solving (word problems)

Under this category (Table 6), questions were formulated to find out whether or not Pre-service mathematics teachers have misconceptions with operations on integers with particular reference to solving word problems. For each of the question in this category, pre-service mathematics teachers were expected to translate the word problem (WP) to an appropriate Mathematics problem and solve accordingly. Bolyard and Moyer-Packenham (2012) also affirm that students had more difficulty creating symbolic representations and making connections among this and other representational forms. Daroczy et al., (2015) claim that recent research shows that a number of linguistic verbal components unrelated to arithmetic make a significant contribution to students' difficulties with word problems is thus affirmed. Again, for some pre-service mathematics teachers to provide numerical responses to word problems that demanded words or a sentence confirms that trainee's misconceptions in operations on integers is rooted on the problem of language acquisition as Makonye and Fakude (2016), opined that major factor serving as an obstacle for students' poor performance in directed numbers is poor command of students over the English Language. Majority of the pre-service mathematics teachers who could not read, understand and solve the word problems relied on guess as stated (Tiwari & Fatima, 2019). Fadillah and Susiaty (2019) claim that some students made mistakes in interpreting the word problems is confirmed.

Table 6: Mean percentage errors in the problem solving (word problems)

Question number	**Test item**	**Number of incorrect responses**	**Percentage error**	**Means percentage error**
Question 8	Subtract – 4 from 5	48	80.00%	
Question 9	Find the sum of 5 and -2	25	41.67%	
Question 10	The temperatures of two drugs in two refrigerators A and B respectively are -100 and -180 which of the two drugs is cooler?	47	78.33%	66.67%

In Table 6, it could be observed from the pre-service mathematics teachers' scripts that some wrote either $-10°$ or $-18°$ as a solution to the word problem. The temperatures of two drugs in two refrigerators A and B respectively were $-10°$ or $-18°$; which of the two drugs is cooler? Instead of writing either drug in refrigerator A or drugs in refrigerator B. This confirms that most of the pre-service mathematics teachers did not read to understand the demands of the problem before finding solutions to the problem but resulted to guessing. Again, for 78.33% of the pre-service mathematics teachers to choose drugs in refrigerator A with temperature of $-10°$ to be cooler than drugs in refrigerator B with temperature of $-18°$ affirms that the pre-service mathematics teachers have misconceptions with integers. Mean percentage error of 66.67% as shown on Table 6, falls under the high misconception group which proves that most of the pre-service mathematics teachers have misconceptions with solving word problems.

E. Errors in the interpretation of absolute value of integers

In Table 8, pre-service mathematics teachers' interpretation of the absolute bar sign was analyzed. The purpose of selecting test items 11 and 18 for this theme was to find out whether or not pre-service mathematics teachers have misconceptions with regards to absolute value of integers. For question 11, "What is the absolute value of -5?", forty-nine (49) out of sixty (60) of the pre-service mathematics teachers provided wrong answers, whereas fifty-nine (59) out sixty (60) of the pre-service mathematics teachers provided wrong answers to question 18, "How will you interpret $|-3|$?"

Table 7: Mean percentage errors in the interpretation of absolute value of integers

Question number	Test item	Number of incorrect responses	Percentage error	Means percentage error
Question 11	What is the absolute value of -5?	49	81.67%	
Question 18	How will you interpret $|-3|$?	59	98.33%	90.00%

Table 7 shows that pre-service mathematics teachers who could not identify the absolute value of -5 had a very high percentage error of 81.67%. As shown in table 8, the majority of pre-service mathematics teachers provided erroneous answers to question 11. According to the summary, majority of the pre-service mathematics teachers did not attempt this question. Moreover, Table 7 shows that fifty-nine (59) of the pre-service mathematics teachers, with a very high percentage error of 98.33% were not table to provide correct interpretation for $|-3|$. This is an indication that only one (1) out of the sixty (60) pre-service mathematics teachers was able to interpret $|-3|$ correctly. Students' interpretations of absolute values of integers, such as $|-3|$ means -3 is an integer and parallel line of -3 confirms Makonye and Fakude's (2016) claim that mathematical understanding of procedures affects practical application and capacity to use mathematical notation, and that learners' errors were caused by their relative inexperience with mathematical logic in this particular instance, learners only mentioned the similarity of 9 and – 9 their absolute value $|9|$.

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F. Errors in problem involving change in the order of operation of integers

Table 8 shows the errors that are made by pre-service mathematics teachers when problems or questions on integers are given whereas they are to change the order of operation. Sahat et al., (2018) claim that most students performed well with problems involving addition integers is affirmed. However, when either the operation signs or the integers change their positions in a mathematics sentence, pre-service mathematics teachers find it difficult to recognize that it is the same mathematics sentence. For example: $-1 + 3 = 3 + (-1)$ is a true sentence but $5 + (-3) = 3 + (-5)$ is not true.

Table 8: Mean percentage errors in problems involving change in the order of operations of integers

Question number	Test item	Number of incorrect responses	Percentage error	Means percentage error
Question 19	Tick (√) in the box to indicate whether or not the statements is true or false: $5 + (-3) = 3 + (-5)$ □ T rue □ False	32	53.33%	
Question 20	Tick (√) in the box to indicate whether or not the statements is true or false: $-1 + 3 = 3 + (-1)$ □ True □ False	18	30.00%	41.67%

In Table 8, questions 19 and 20 were formulated as True or False questions to test whether or not pre-service mathematics teachers would be able to identify that addition and subtraction problems involving integers will remain the same even if the operations or the integers change positions. The pre-service mathematics teachers were provided with an empty box to tick their preferred option to the test items. Table 8 indicates that thirty-two (32) of the pre-service mathematics teachers (53.33%) wrongly selected True as the best option for $5 + (-3) = 3 + (-5)$ whereas eighteen (18) of the pre-service mathematics teachers (30.00%) wrongly choose False for test $-1 + 3 = 3 + (-1)$. The mean percentage error of 41.67% gives credence to the fact that pre-service mathematics teachers have challenges with that the commutative proper of addition of integer.

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G. Errors in solving problems involving more than one operation

In Table 9, errors in solving problems involving more than one operation of the Pre-service mathematics teachers were discussed.

Table 9: Mean percentage errors in solving problems involving more than one operation

Question number	Test item	Number of incorrect responses	Percentage error	Means percentage error
Question 7	Evaluate $(-2)-(-8)\times(+2)$	58	96.67%	96.67%

Table 9 shows that as many as fifty-eight (58) of the pre-service mathematics teachers representing 96.67% which is categorized as very high failed to answer question 7, "*Evaluate* $(-2)-(-8)\times(+2)$", correctly. From the study, it could be seen that most of the pre-service mathematics teachers had problems with evaluation of (–2) – (–8) which resulted in most of the pre-service mathematics teachers answering (–2) – (–8) × (+2) wrongly. In the item analysis of this error, some of the pre-service mathematics teachers considered the subtraction part of the problem, "$(-2)-(-8)$" before considering the multiplication part, "$(-8)\times(+2)$". Again, pre-service mathematics teachers solved $(-2)-(-8)\times(+2)$ by first multiplying negative two (-2) by positive eight $(+8)$. That is, $(-2)\times(+8)$ to get negative sixteen (-16). Another trainee solved $(-2)-(-8)\times(+2)$ by considering $(-2)-(-8)$ as $8+2$. She had 10. When went on to multiply by 2 to get 20. The findings of Rosyidah, Maulyda, Jiwandono, Oktaviyanti, Gunawan (2021), that students do not comprehend the laws and guidelines in the completion of positive and negative integers, with the result that students occasionally overlook the negative or positive symbols of integers, assuming there is no distinction between the two. Furthermore, Jabal and Rosjanuardi (2019) agree that students give information without following proper procedure of operation, and their successive supposition that students lack knowledge of the rules of operation of numbers, which is, if there is a question containing multiplication, subtraction, division and additions, the transactions should be done first, namely multiplication, division, addition, and subtraction, is confirmed by this study.

H. Mean percentage errors in ordering a set of integers

In Table 10, question 17 was formulated to find out whether or not pre-service mathematics teachers' have misconceptions with cardinality of integer. Table 10 shows that 40 of the pre-service mathematics teachers (66.67%), could not determine the cardinality of the integers which influence their incorrect arrangement of the integers.

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Table 10: Ordering a set of integers

Question number	Test item	Number of incorrect responses	Percentage error	Mean percentage error
Question 17	Arrange the following numbers in increasing order of magnitude: $-5, 3, 0, -8, -12$	40	66.67%	66.67%

The concept of ordering is a fundamental principle of our number system, and children's first encounters with it occur when they learn to count and reason about before and after, as well as smaller and greater (Bishop et al., 2014). However, the study findings contradict this statement because a greater number of Pre-service mathematics teachers (adults being trained to teach children) have difficulty ordering a set of integers. A higher percentage error of 66.67% indicating that majority of the Pre-service mathematics teachers fail to properly arrange the following $-5, 3, 0, -8, -12$ integers in increasing order of magnitude supports Makonye and Fakude (2016) declaration that students seem to know that the negative numbers are smaller 71 than the positive numbers. But the challenge seems to be where they have to decide which is bigger or smaller one among the negative numbers themselves.

General Performance (In Errors) for each Theme

Table 11 indicates that among the errors identified under each theme, errors in addition and subtraction of integers (27.22%) is the least recorded. This suggests that, although Pre-service mathematics teachers have misconceptions with regards to addition and subtraction of integers, the number of pre-service mathematics teachers with these misconceptions are relatively low. The pean percentage error of 31.53% for the basic operations on integers (addition, subtraction, multiplication and division), even though is categorized as very low is worrying since this operations serve as the bases for which other complex operations on integers to be built upon.

Table 11: General performance in errors for each theme

Themes (on errors)	Mean percentage error
Errors in Interpretation of the inequality symbols '< and >'	62.77%
Errors in Addition and Subtraction of Integers	27.22%
Errors in Multiplication and Division of Integers	35.83%
Errors in Problem solving (word problems)	66.67%
Errors in Interpretation of absolute value of integers	90.00%
Errors in Problems involving Change in the order of operation of integers	41.67%
Errors in Solving Problems involving more than one operation	96.67%
Errors in Ordering a set of integers	66.67%
Overall Mean Percentage Error	60.94%

Again, in Table 11, mean percentage error of 96.67% for errors in solving problems involving more than one operation clearly show that pre-service mathematics teachers have much greater misconceptions in in solving problems involving more than one operation. Also, mean percentage error in interpreting the absolute value of integers (90.00%) is the second highest among the common error of pre-service mathematics teachers which indicates that most of the pre-service mathematics teachers have difficulty in dealing with the concept of absolute value of integers. The overall mean percentage error of 60.94% is an indication that averagely each Training in one way or other has a misconception relating to operations on integers. The common errors of pre-service mathematics teachers were computed for and presented in Table 11 to compere how the pre-service mathematics teachers fared under each of the categories of common errors. The errors of pre-service mathematics teachers in order of severity as presented in Table 15 are ranked as follows: errors in solving problems involving more than one operation was 96.67%. Also, errors in interpretation of absolute value of integers was 90.00%. The errors in problem solving (word problems) or errors in ordering a set of integers was 66.67%. Similarly, errors in interpretation of the inequality symbols '< and >' was 62.77%. The errors in problems involving change in the order of operation of integers was 41.67. The errors in multiplication and division of integers was 35.83%) and errors in addition and subtraction of integers was 27.22%. It is clearly shown that among the various themes of classifying pre-service mathematics teachers' errors on operations on integers, errors in solving problems involving more than one operation stands tall. Meaning majority of the pre-service mathematics teachers' misconceptions are shown in problems involving more than one of the basic operations (addition, subtraction, multiplication and division) of integer whereas the errors in addition and subtraction of integers has the least number of occurrences.

Table 12: Independent sample t-test comparing misconceptions of early grade and upper primary pre-service mathematics teachers

	F	Sig.	t	df	Sig. (2-tailed)	Mean difference	Std. Error difference
Early grade	.409	.528	-.884	28	.384	-.533	1.735
Upper primary			-.884	27.852	.384	-1.533	1.735

Table 12 shows the Levene's test for equality of variance with p = 0.528 meaning variances across the two groups are not statistically different from each other. The independent sample t-TEST between the performance of early grade and upper primary students of the study college showed that there is no significance difference between their performances. This implies that pre-service mathematics teachers of both specialisms have equal levels of misconception of operations on integers. Again, Table 16 that the significant value is greater than 0.05, that is for the independent samples t-test, $t\,(28) = -.884$, p $.384 > 0.05$. This suggests that, there is no statistically significant difference between early grade and upper primary regarding their misconception.

Interview results of reasons that accounted for pre-service mathematics teachers' erroneous answers

Results of the interview revealed that most of the pre-service mathematics teachers' misconceptions were as a result of how the concept of integers was introduced to them at basic school. Trainee coded EG 05 (early grade) indicated that, "the Basic School Teacher used left upper and lower arms burned to introduce the inequality symbol (<) as less than whereas the right upper and lower arms burned was used to introduce the inequality (>) as greater than". Again, the interview showed that some of the pre-service mathematics teachers learned the concept of integers as fix rules that needed to be memorized and applied. Trainee coded PE (upper primary) 15 stated that: *"when two negative numbers meet it becomes positive thus negative-negative positive".* The interview results revealed that most pre-service mathematics teachers did not apply BODMAS when solving problems of integers involving more than one operation signs. Pre-service mathematics teachers coded PE 10 and EG 05 admitted that even though they have heard of the acronym BODMAS for evaluating integers problem involving more than two operations, they could not apply it to solve the problems appropriately. Trainee coded EG 02 explained that; "to solve $(-2) - (-8) \times (+2)$, I first multiplied 2 by 8 to get 16 which was then multiplied by 2 to get 32".

CONCLUSION

The study concluded that the following findings: interpretation of the inequality symbols $"<"$ and $">"$, addition and subtraction of integers, multiplication and division of integers, problem solving (word problems), interpretation of absolute value of integers, problems involving change in the order of operation 75 of integers, problems involving more than one operation and ordering a set of integers are the common errors trainee's make on operations on integers. The study has also brought to light that most of the errors pre-service mathematics teachers commit when performing operations on integers. The errors included how the concepts of operations on integers were introduced to pre-service mathematics teachers at the basic school level; deficiency of pre-service mathematics teachers in the English Language, carelessness of the pre-service mathematics teachers (ignoring the signs in front of the integers), rote-learning (leading to misinterpretation and guessing), and over-reliance on calculator usage. Again, the study has made it clear that pre-service mathematics teachers of both the early specialism and the upper primary specialism equally have misconceptions on operations on integers.

RECOMMENDATIONS

In light of the study findings, it is recommended that when solving word problems involving integers, pre-service mathematics teachers should take their time and read over the problem thoroughly in order to understand the demands of the problem. Also, during teaching and learning of mathematics particularly operations on integers, much attention must be given to helping learners to translate the everyday language to a mathematics language since language deficiency was found to be one of the major causes of pre-service mathematics teachers misconceptions.

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Conflict of Interest

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Authors Contributions

Conceptualization was done by Christopher and Alexander, data collection was done by Alexander, Dennis and Boare, data analysis was done by Alexander, Christopher and Dennis,

methodology was done by Alexander and Boare, original draft preparation was done by Dennis and Boare while supervision was done by Christopher.

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